Reduction Using Induced Subnets To Systematically Prove Properties For Free-Choice Nets

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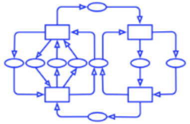

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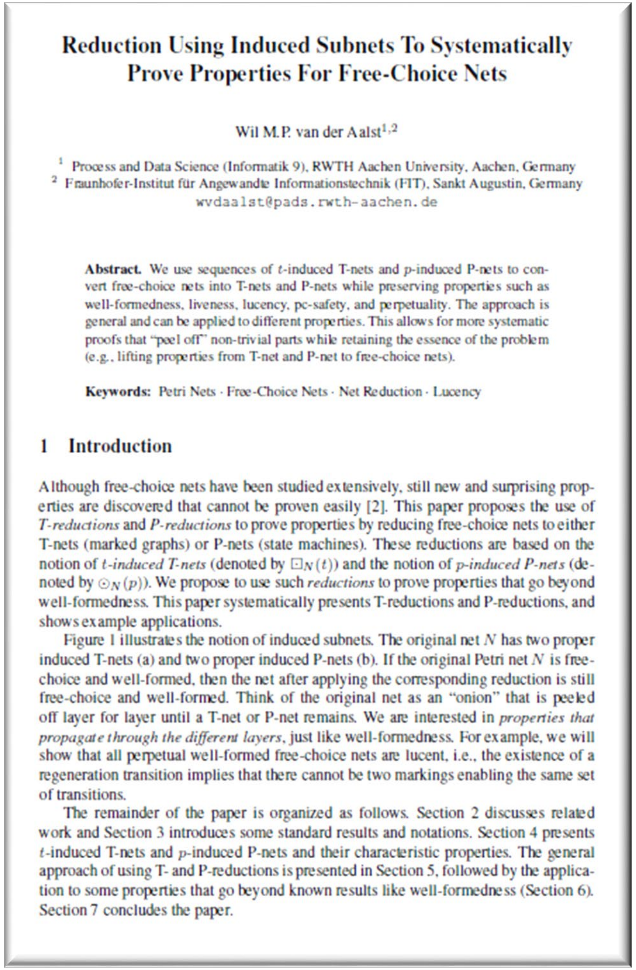

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Wil M.P. van der Aalst[1,2]

[1] Process and Data Science (Informatik 9), RWTH Aachen University, Aachen, Germany
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Abstract. We use sequences of t-induced T-nets and p-induced P-nets to convert free-choice nets into T-nets and P-nets while preserving properties such as well-formedness, liveness, lucency, pc-safety, and perpetuality. The approach is general and can be applied to different properties. This allows for more systematic proofs that "peel off" non-trivial parts while retaining the essence of the problem (e.g., lifting properties from T-net and P-net to free-choice nets).

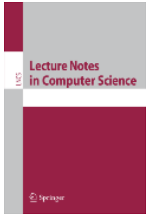
Keywords: Petri Nets · Free-Choice Nets · Net Reduction · Lucency

1 Introduction

Although free-choice nets have been studied extensively, still new and surprising properties are discovered that cannot be proven easily [2]. This paper proposes the use of *T-reductions* and *P-reductions* to prove properties by reducing free-choice nets to either T-nets (marked graphs) or P-nets (state machines). These reductions are based on the notion of *t-induced T-nets* (denoted by $\boxminus_N(t)$) and the notion of *p-induced P-nets* (denoted by $\odot_N(p)$). We propose to use such *reductions* to prove properties that go beyond well-formedness. This paper systematically presents T-reductions and P-reductions, and shows example applications.

Figure 1 illustrates the notion of induced subnets. The original net N has two proper induced T-nets (a) and two proper induced P-nets (b). If the original Petri net N is free-choice and well-formed, then the net after applying the corresponding reduction is still free-choice and well-formed. Think of the original net as an "onion" that is peeled off layer for layer until a T-net or P-net remains. We are interested in *properties that propagate through the different layers*, just like well-formedness. For example, we will show that all perpetual well-formed free-choice nets are lucent, i.e., the existence of a regeneration transition implies that there cannot be two markings enabling the same set of transitions.

The remainder of the paper is organized as follows. Section 2 discusses related work and Section 3 introduces some standard results and notations. Section 4 presents t-induced T-nets and p-induced P-nets and their characteristic properties. The general approach of using T- and P-reductions is presented in Section 5, followed by the application to some properties that go beyond known results like well-formedness (Section 6). Section 7 concludes the paper.



Preprint available: Reduction Using Induced Subnets To Systematically Prove Properties For Free-Choice Nets. CoRR abs/2106.03658 (2021).

Fundamenta Informaticae

Like this work? Also consider reading the paper Wil van der Aalst: Free-Choice Nets With Home Clusters Are Lucent. CoRR abs/2106.03554 (2021). Accepted for Fundamenta Informaticae (in print).

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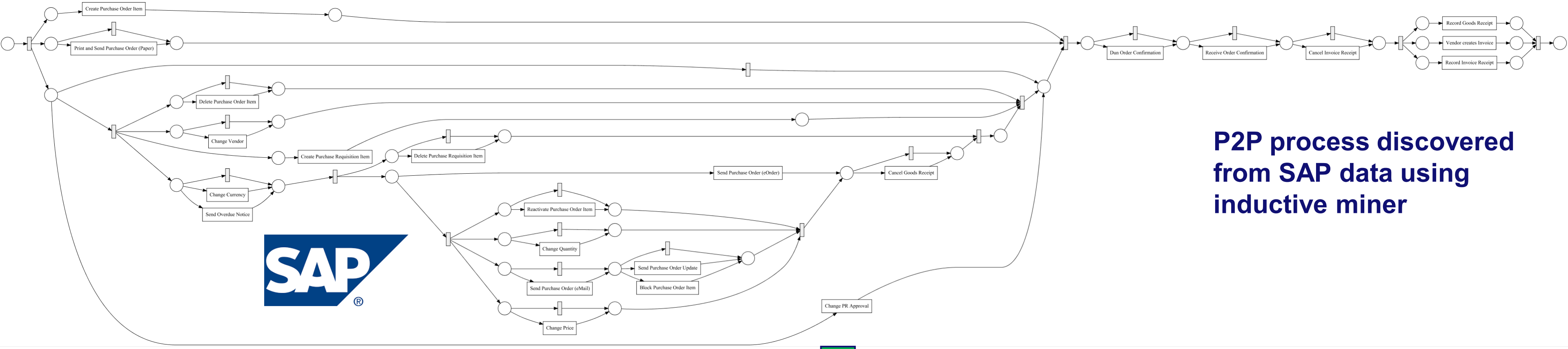
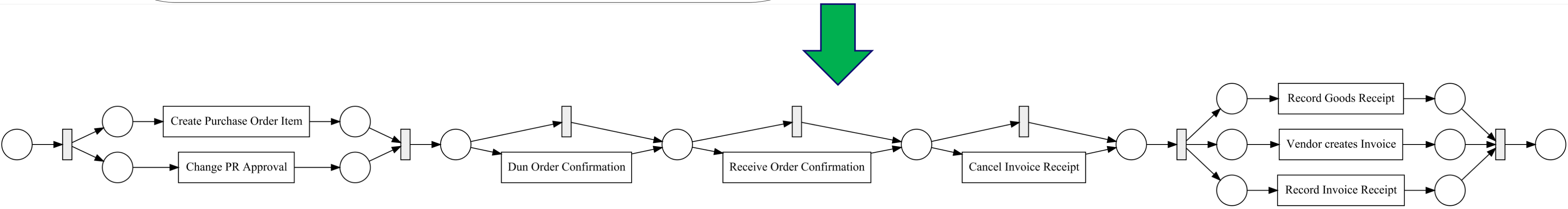
Reductions



P2P process discovered from SAP data using inductive miner



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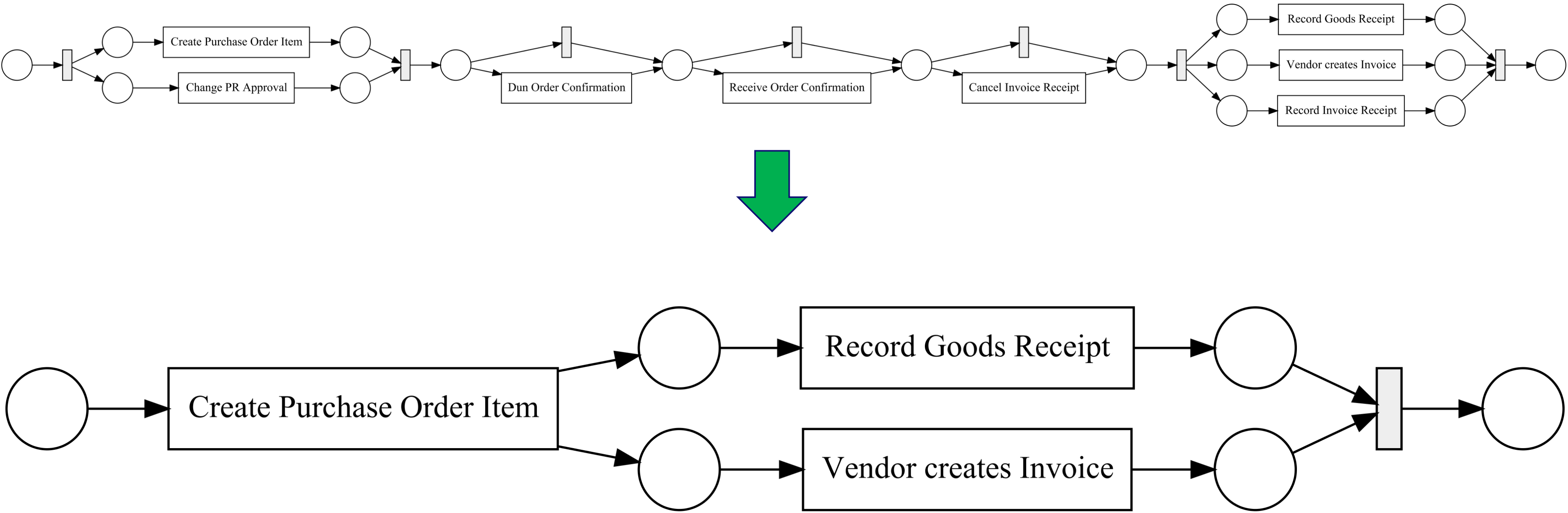
Reductions

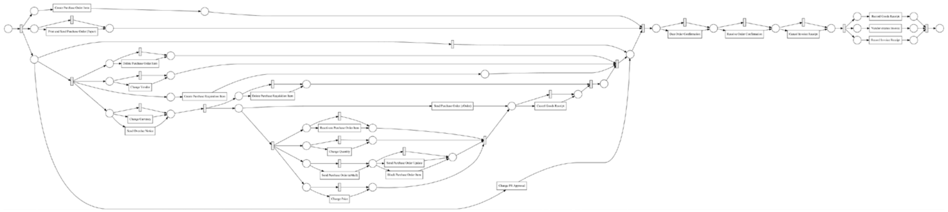


Create Purchase Order Item
Change PR Approval
Dun Order Confirmation
Receive Order Confirmation
Cancel Invoice Receipt
Record Goods Receipt
Vendor creates Invoice
Record Invoice Receipt
Create Purchase Order Item
Record Goods Receipt
Vendor creates Invoice

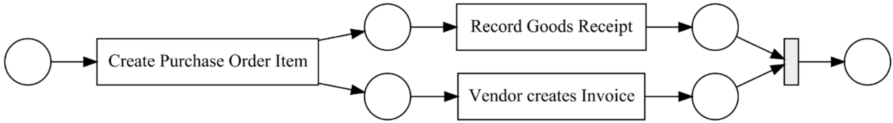
Reductions

short-circuit nets if workflow nets

(org)



(red)



X is preserved downstream: If (org) is X, then (red) is X.

X is preserved upstream: If (red) is X, then (org) is X.

Typical candidates for X: live, bounded, well-formed, free-choice, lucent, deadlock free, safe, etc.

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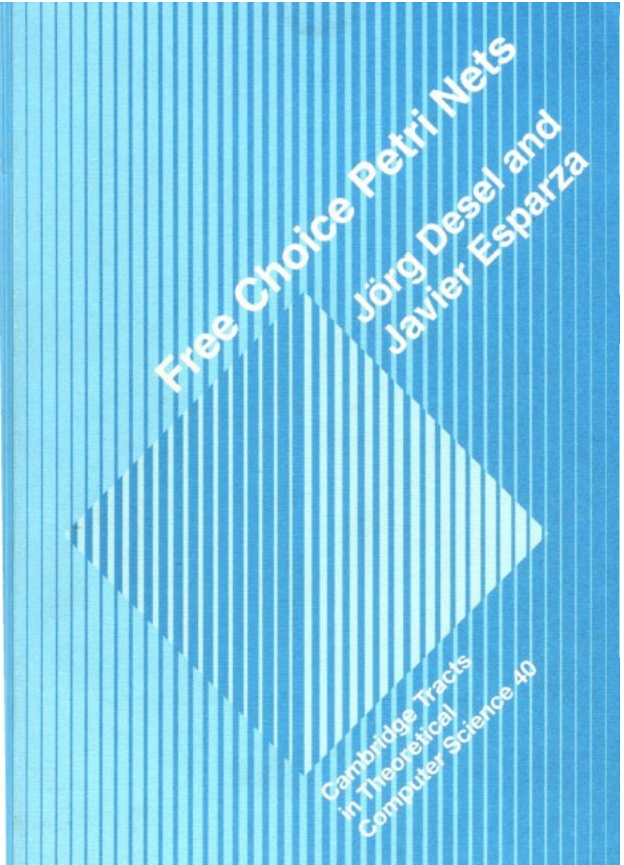
Well-known example: Free-choice nets

(see also work of Berthelot, Genrich, Thiagarajan, Murata, Kovalyov, Verbeek, Wynn, etc.)



Rule 1 *The rule* ϕ_A

Let N *and* $\tilde{N}$ *be two free-choice nets, where* $N = (S,T,F)$ *and* $\tilde{N} = (\tilde{S},\tilde{T},\tilde{F})$. $(N,\tilde{N}) \in \phi_A$ *if there exist a place* $s \in S$ *and a transition* $t \in T$ *such that:*

Conditions on N:

1. ${}^\bullet s \neq \emptyset$, $s^\bullet = \{t\}$
2. $t^\bullet \neq \emptyset$, ${}^\bullet t = \{s\}$
3. $({}^\bullet s \times t^\bullet) \cap F = \emptyset$

Construction of $\tilde{N}$:

4. $\tilde{S} = S \setminus \{s\}$
5. $\tilde{T} = T \setminus \{t\}$
6. $\tilde{F} = (\,F \cap ((\tilde{S} \times \tilde{T}) \cup (\tilde{T} \times \tilde{S}))\,) \cup ({}^\bullet s \times t^\bullet)$

(where the dot-notation is taken with respect to N*).*

Rule 2 *The rule* ϕ_S

Let N *and* $\tilde{N}$ *be two free-choice nets.* $(N,\tilde{N}) \in \phi_S$ *if:*

Conditions on N:

1. *N contains at least two places*
2. *N contains a nonnegative linearly dependent place s*
3. ${}^\bullet s \cup s^\bullet \neq \emptyset$, *i.e., s is not an isolated place*

Construction of $\tilde{N}$:

4. $\tilde{N} = N \setminus \{s\}$

Rule 3 *The rule* ϕ_T

Let N *and* $\tilde{N}$ *be two free-choice nets.* $(N,\tilde{N}) \in \phi_T$ *if:*

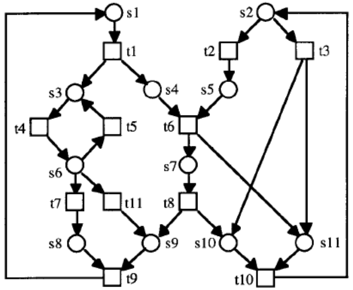
Conditions on N:

1. *N contains at least two transitions*
2. *N contains a nonnegative linearly dependent transition t*
3. ${}^\bullet t \cup t^\bullet \neq \emptyset$, *i.e., t is not an isolated transition*

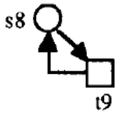
Construction of $\tilde{N}$:

4. $\tilde{N} = N \setminus \{t\}$

(org)



(red)



Proof uses the notion of CP-nets which is related to T-reductions in this paper

(org) is well-formed if and only if (red) is well formed.
Moreover, any well-formed free choice net can be reduced to the atomic net.

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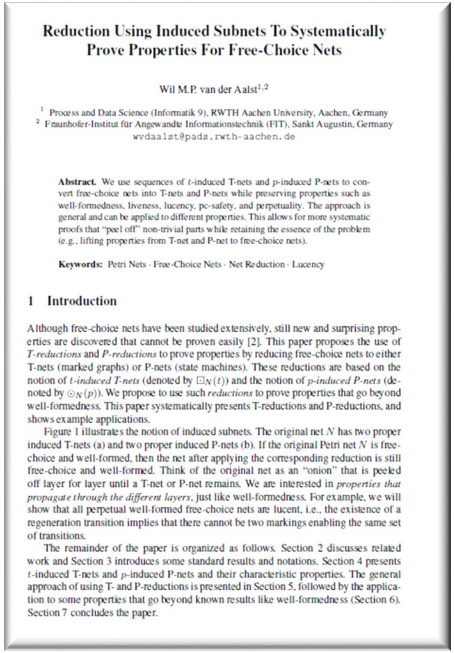
This paper

- *t*-induced T-net (related to CP-nets)
- *p*-induced P-net (new concept)
- Existence of reductions (always two until T- or P-net)
- Preserves free-choice, well-formedness, liveness, boundedness, pc-safeness, and *perpetuality* “downstream”.
- Preserves *lucency* “upstream” (assuming perpetuality)
- Generic framework



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t-induced T-net

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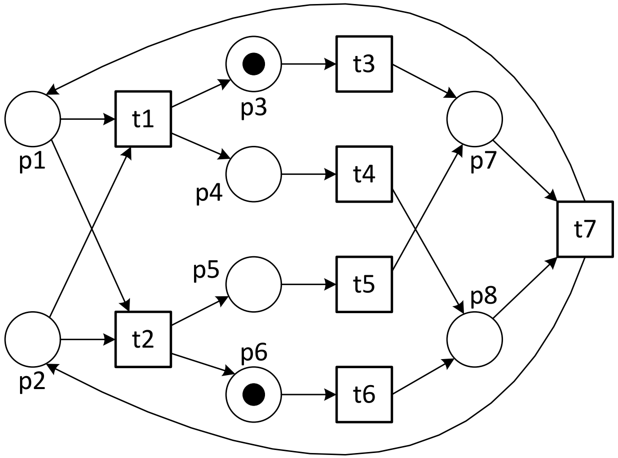
t-induced T-net



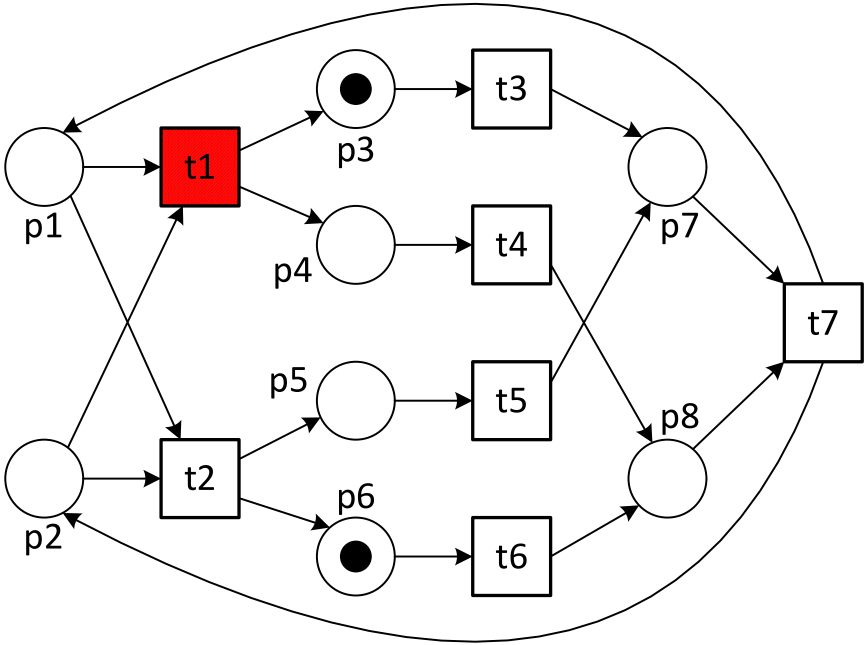
Definition 10 **(t-Induced T-net).** *Let* $N = (P, T, F)$ *be a Petri net and* $t \in T$. $\boxdot_N(t) \subseteq P \cup T$ *is the smallest set such that*

- $t \in \boxdot_N(t)$,
- $\{p' \in t'\bullet \mid |\bullet p'| = 1 \ \wedge \ |p'\bullet| = 1\} \subseteq \boxdot_N(t)$ *for any* $t' \in \boxdot_N(t) \cap T$, *and*
- $p'\bullet \subseteq \boxdot_N(t)$ *for any* $p' \in \boxdot_N(t) \cap P$.

$\boxdot_N(t)$ *are the nodes of the t-induced T-net of N that is denoted by* $N_{\boxdot(t)} = N\restriction_{\boxdot_N(t)}$. $\overline{N_{\boxdot(t)}} = N \setminus\!\!\setminus \boxdot_N(t)$ *is the complement of the t-induced T-net of N.* $\boxdot_N(t)$ *is proper if the complement* $\overline{N_{\boxdot(t)}}$ *is a non-trivial strongly-connected Petri net.*

t-induced T-net: Pick a transition *t*

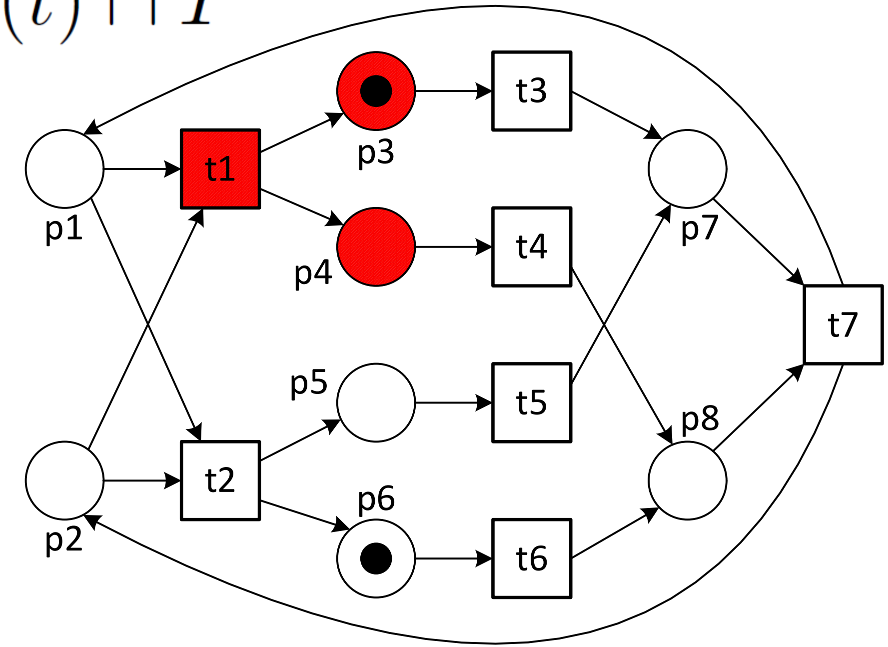
$$t \in \boxdot_N(t)$$



t-induced T-net: Add output places

$$\{p' \in t'\bullet \mid |\bullet p'| = 1 \;\wedge\; |p'\bullet| = 1\} \subseteq \boxdot_N(t)$$

$$\textit{for any } t' \in \boxdot_N(t) \cap T$$

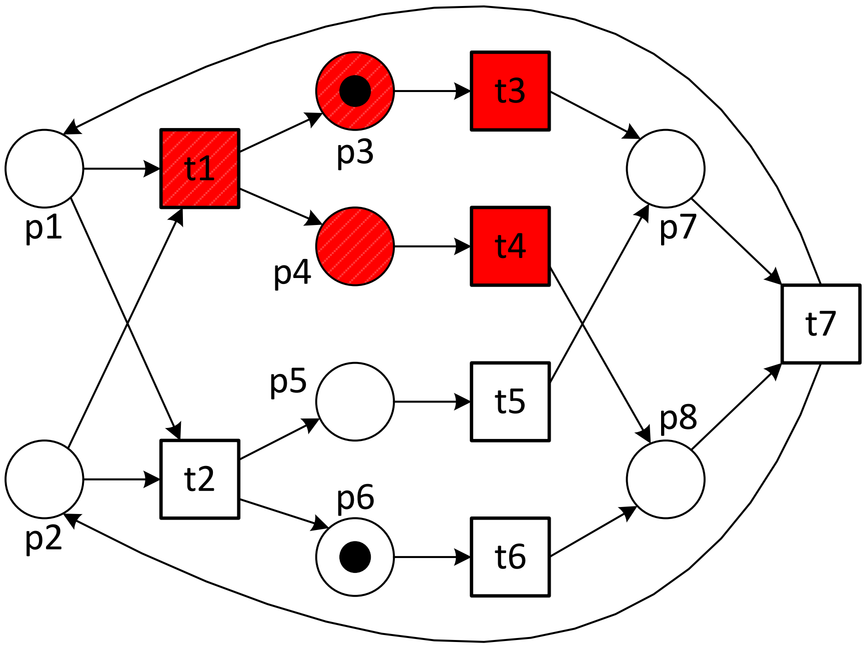


Only add places with one input and one output transition

t-induced T-net: Add output transitions

$$p'\bullet \subseteq \boxdot_N(t) \; for \; any \; p' \in \boxdot_N(t) \cap P$$

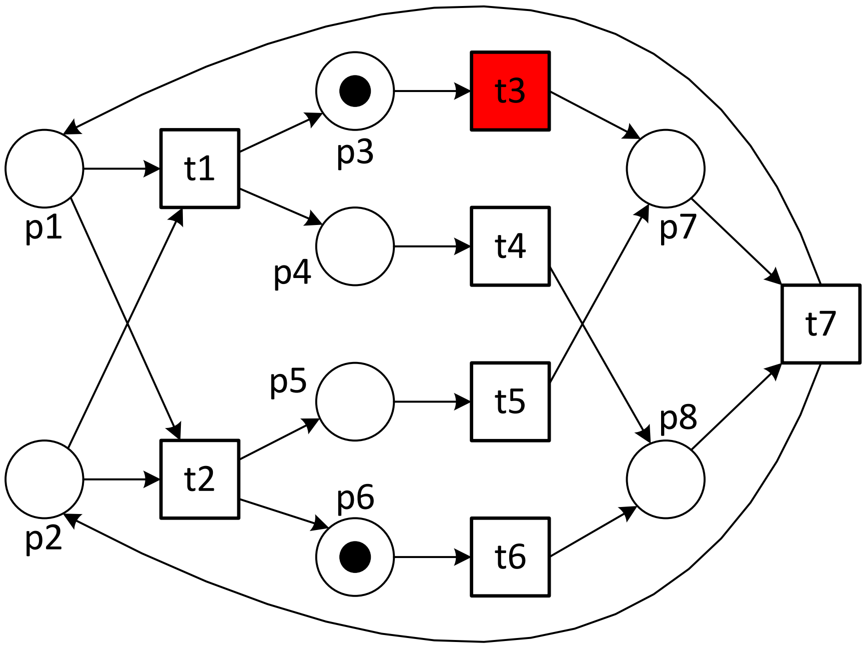
$$\boxdot_N(t1)$$



Cannot be extended further

t-induced T-net: Pick another transition *t*

$$\boxdot_N(t3)$$



Cannot be extended further

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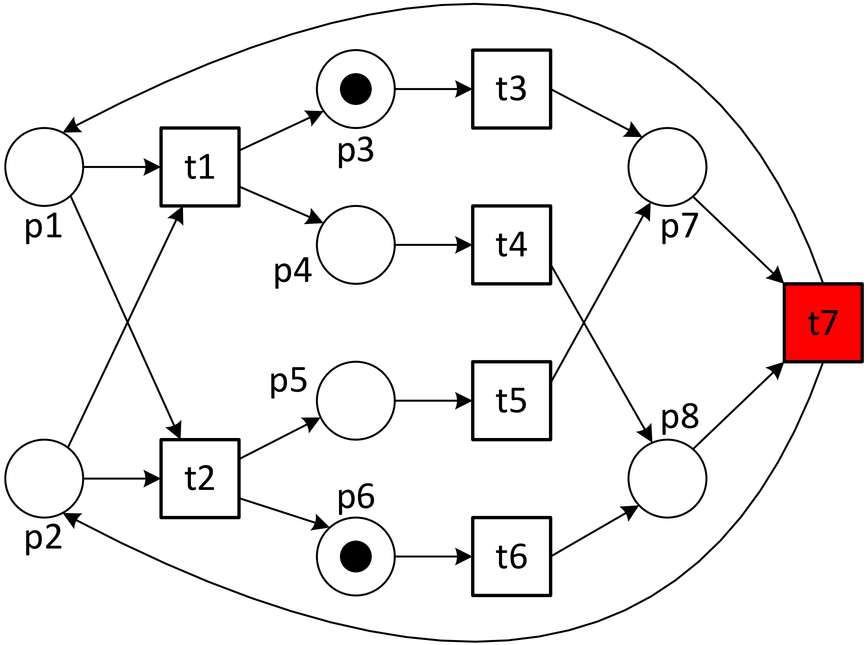
t-induced T-net: Pick another transition *t*

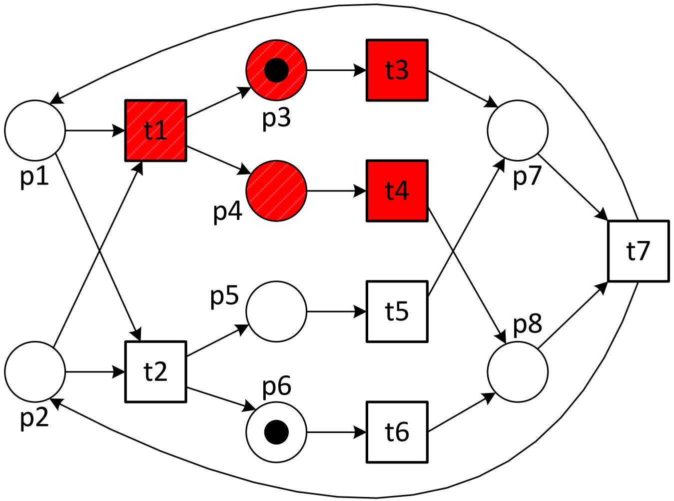
$$\boxdot_N(t7)$$

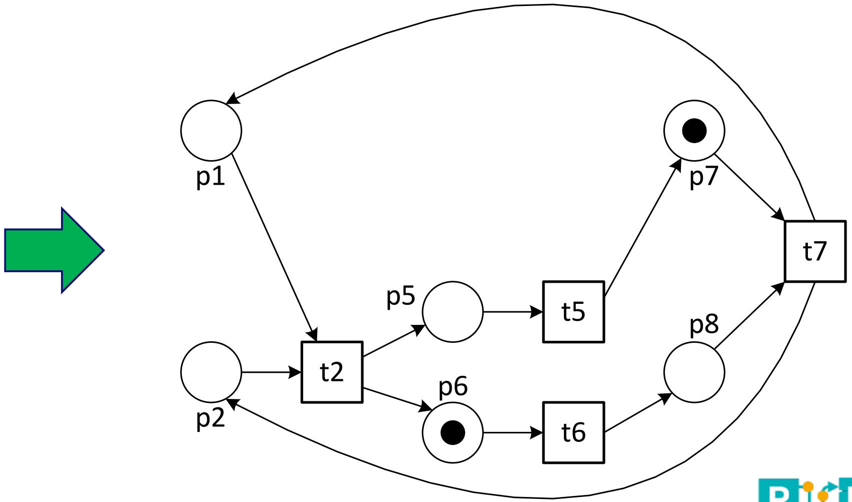


Cannot be extended further

Proper: Complement is strongly connected

$$\boxdot_N(t1)$$

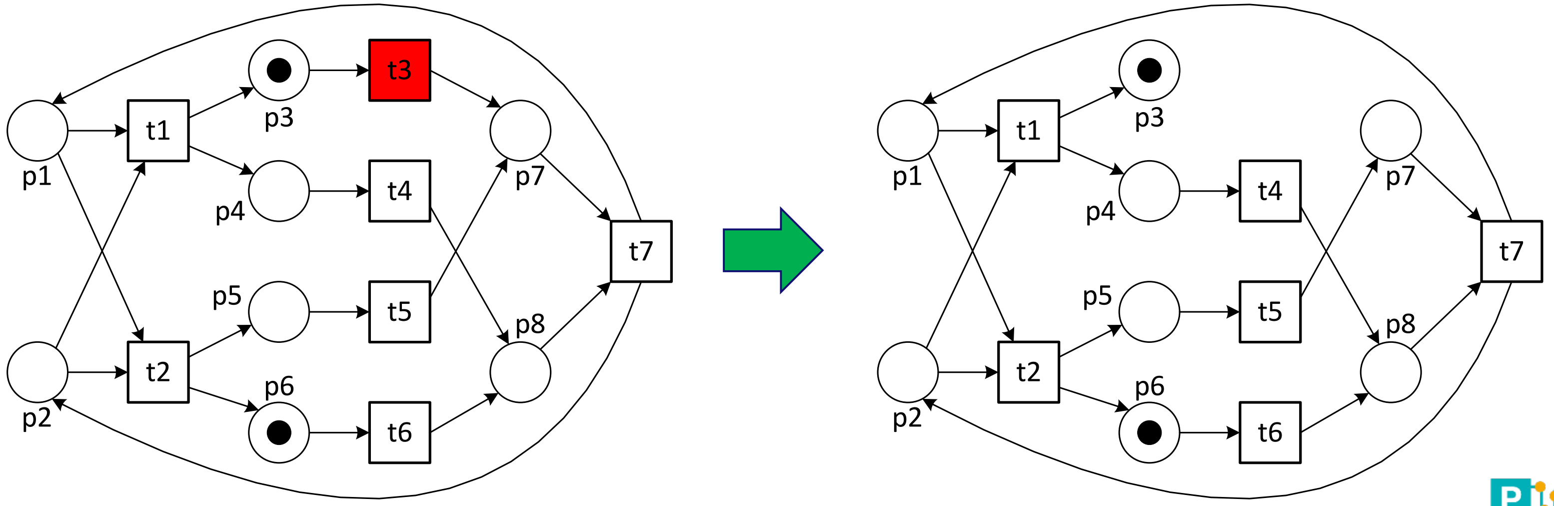




The complement has a new marking see paper for details.

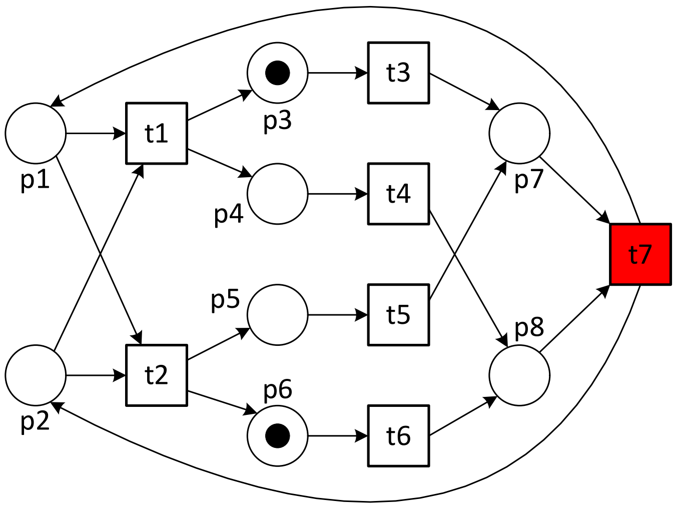
Not proper: Complement is <u>not</u> strongly connected

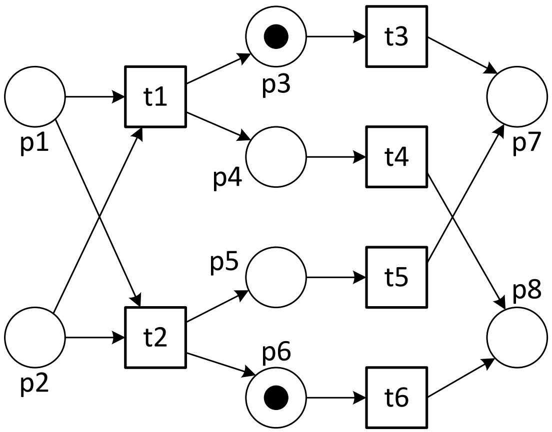
$$\boxdot_N(t3)$$



Not proper: Complement is not strongly connected

$$\boxdot_N(t7)$$





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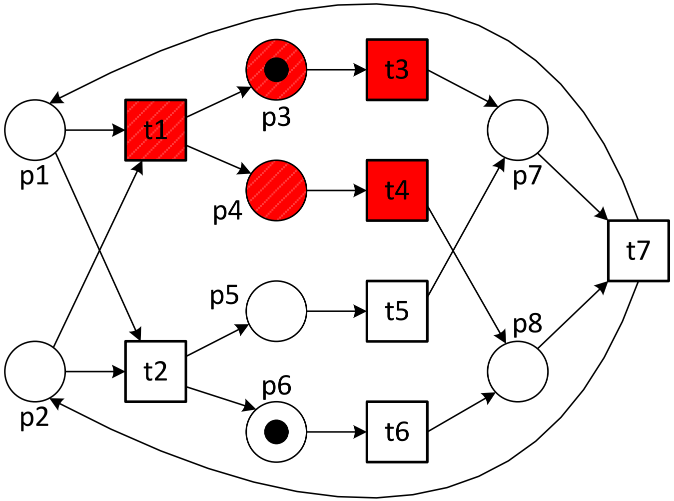
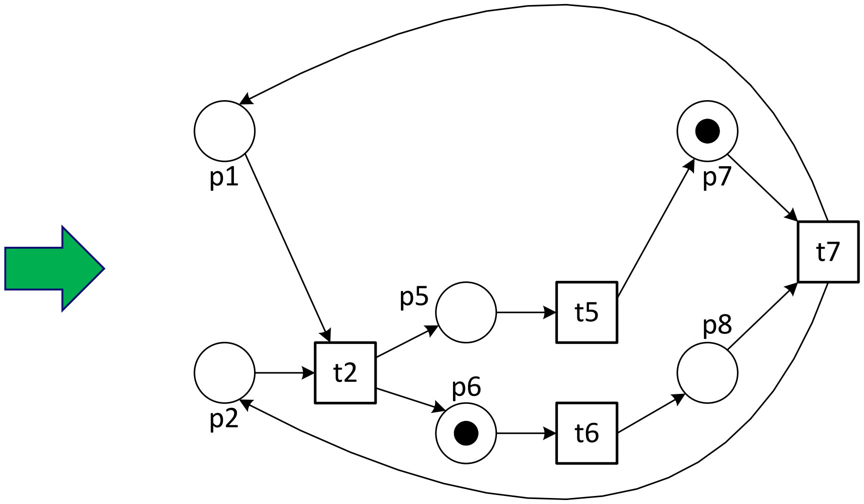
Complement

$$\overline{N_{\boxdot(t)}} = N \setminus\setminus \boxdot_N(t)$$

$$\overline{N_{\boxdot(t1)}} = N \setminus\setminus \boxdot_N(t1)$$

proper *t1* induced T-net

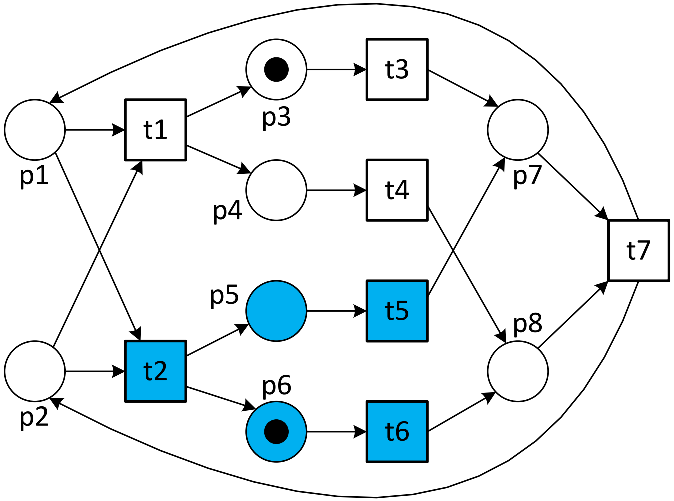




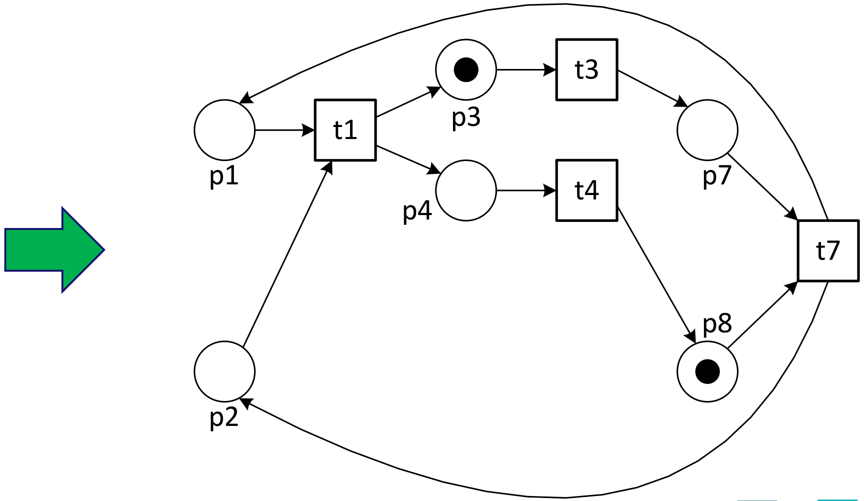
Complement

$$\overline{N_{\boxdot(t)}} = N \setminus\!\setminus \boxdot_N(t)$$

$$\overline{N_{\boxdot(t2)}} = N \setminus\!\setminus \boxdot_N(t2)$$



proper *t2* induced T-net



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p-induced P-net

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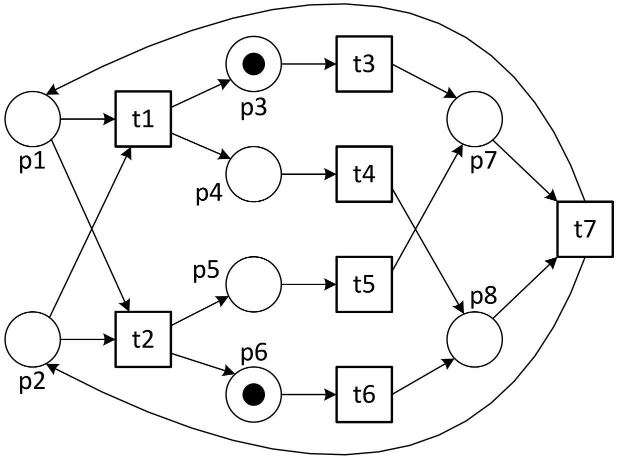
p-induced P-net



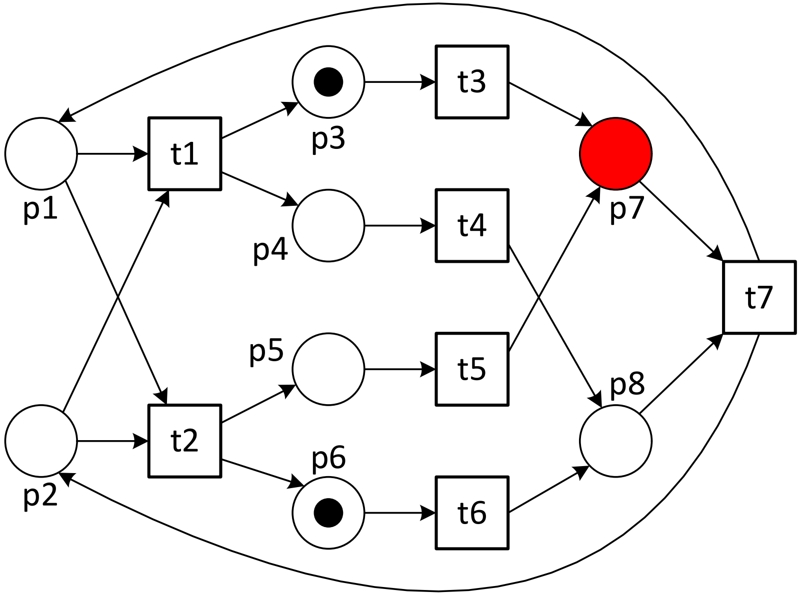
Definition 11 **(p-Induced P-net).** *Let $N = (P, T, F)$ be a Petri net and $p \in P$. $\odot_N(p) \subseteq P \cup T$ is the smallest set such that*

- $p \in \odot_N(p)$,
- $\{t' \in \bullet p' \mid |\bullet t'| = 1 \ \wedge \ |t' \bullet| = 1\} \subseteq \odot_N(p)$ *for any* $p' \in \odot_N(p) \cap P$, *and*
- $\bullet t' \subseteq \odot_N(p)$ *for any* $t' \in \odot_N(p) \cap T$.

$\odot_N(p)$ are the nodes of the p-induced P-net of N which is denoted by $N_{\odot(p)} = N \restriction_{\odot_N(p)}$. $\overline{N_{\odot(p)}} = N \setminus \odot_N(p)$ is the complement of the p-induced P-net of N. $\odot_N(p)$ is proper if the complement $\overline{N_{\odot(p)}}$ is a non-trivial strongly-connected Petri net.

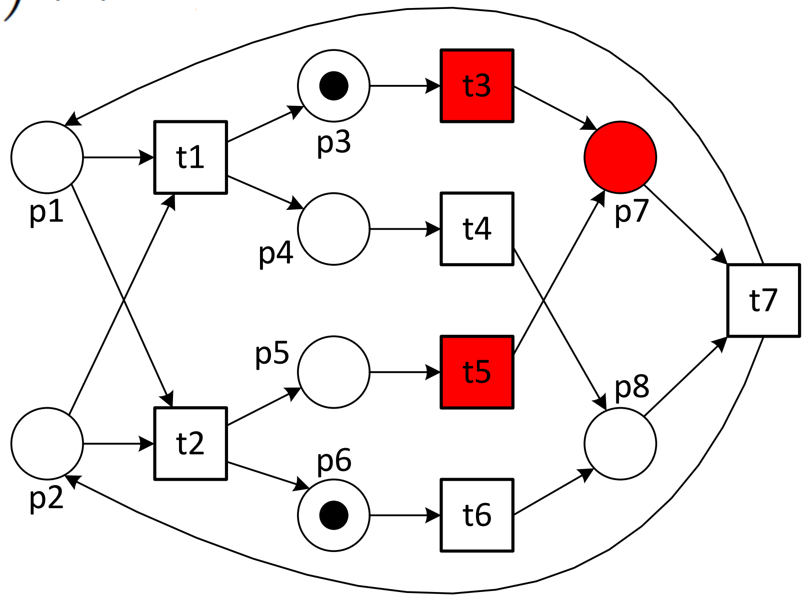
p-induced P-net: Pick a place

$$p \in \odot_N(p)$$



p-induced P-net: Add input transitions

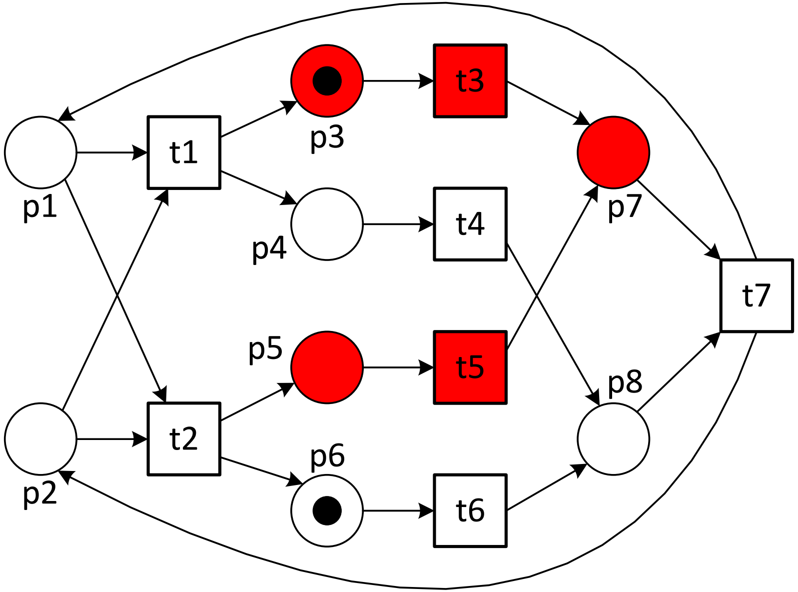
$$\{t' \in \bullet p' \mid |\bullet t'| = 1 \wedge |t'\bullet| = 1\} \subseteq \odot_N(p)$$
$$\mathit{for\ any}\ p' \in \odot_N(p) \cap P$$



Only add transitions with one input and one output place

p-induced P-net: Add input places

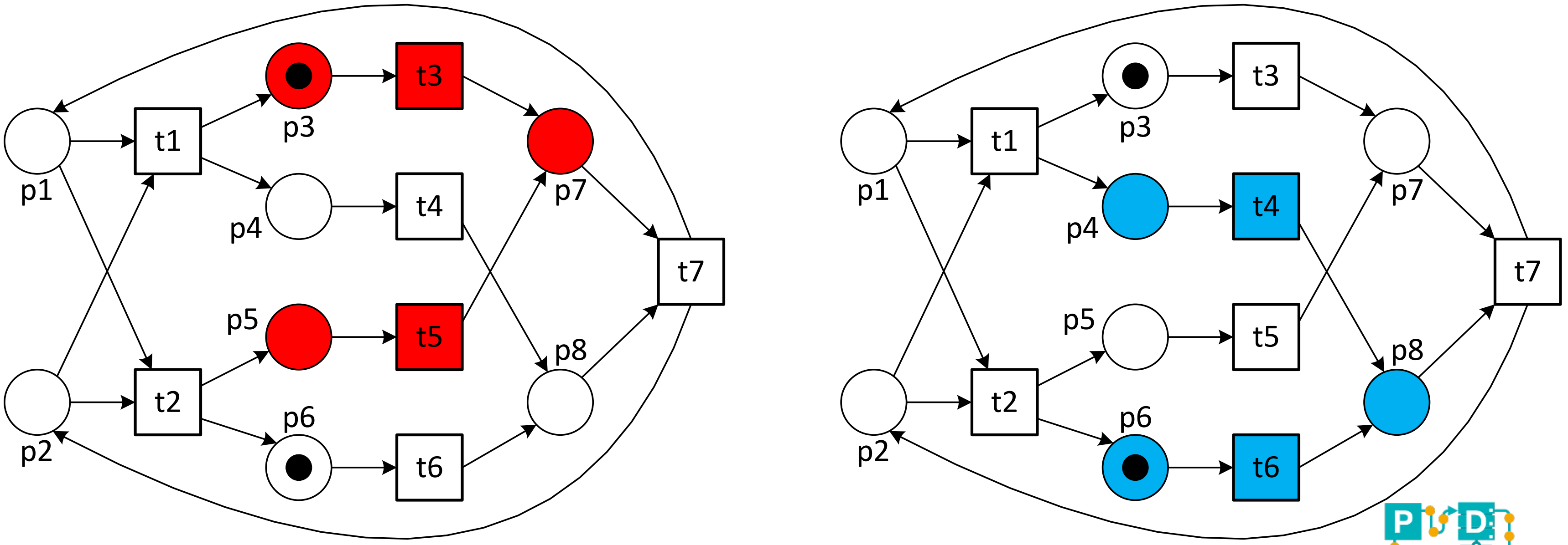
$$\bullet t' \subseteq \odot_N(p) \text{ for any } t' \in \odot_N(p) \cap T$$



Cannot be extended further

Two proper *p*-induced P-net

$\odot_N(p7)$ $\odot_N(p8)$

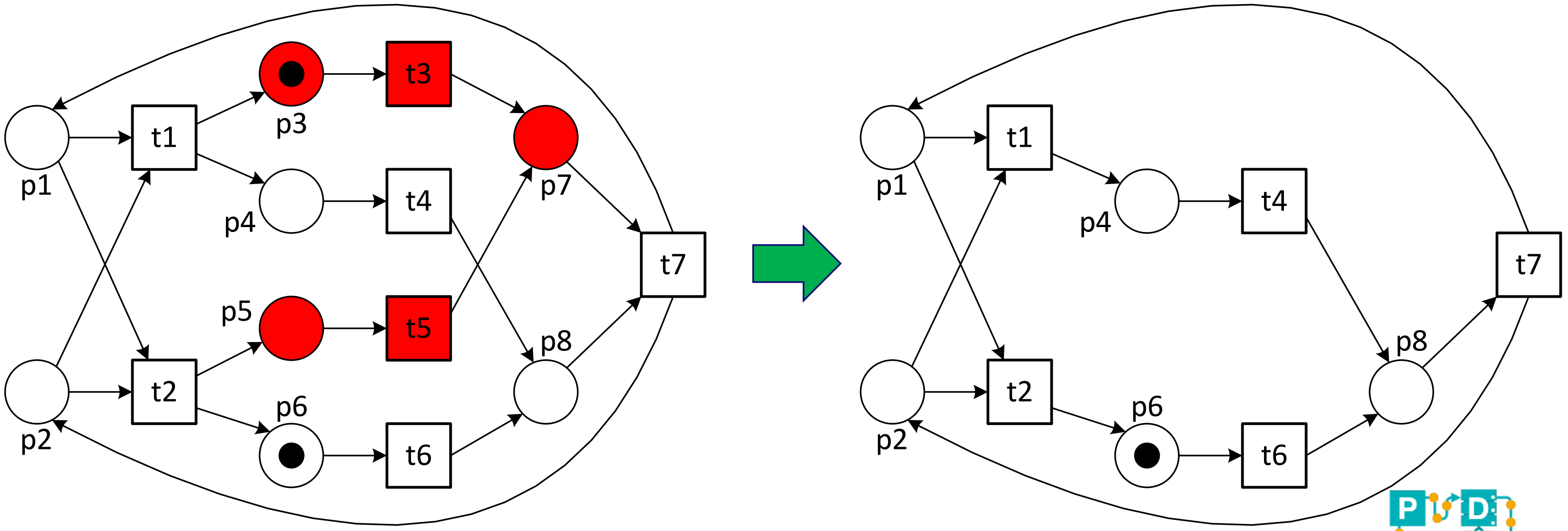


Again proper means that the complement is strongly connected.

Complement of proper *p7*-induced P-net

$$\odot_N(p7)$$

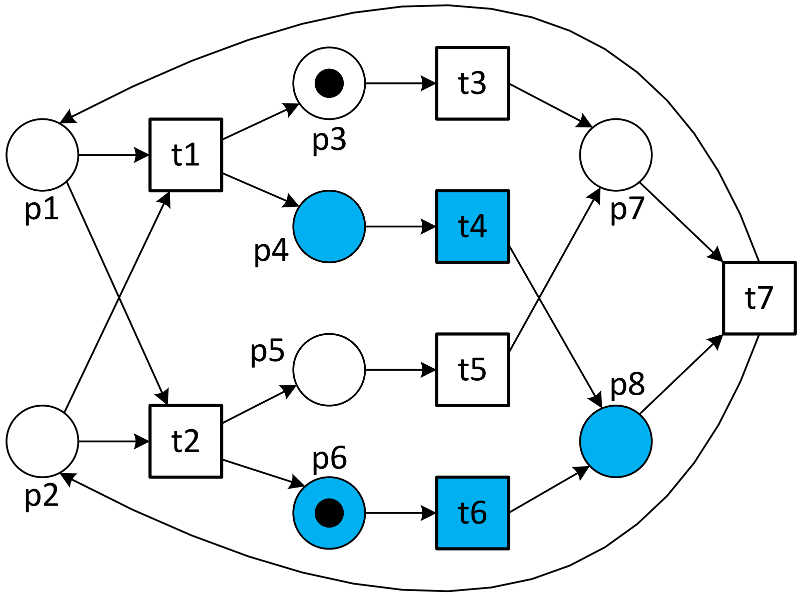
$$\overline{N_{\odot(p7)}} = N \setminus\!\setminus \odot_N(p7)$$

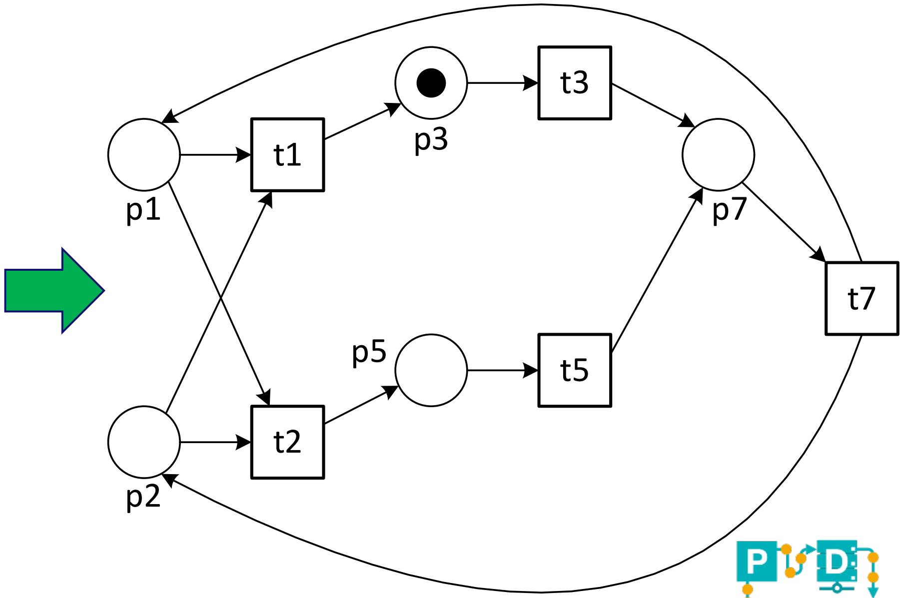


Complement of proper *p8*-induced P-net

$$\odot_N(p8)$$

$$\overline{N_{\odot(p8)}} = N \setminus\!\setminus \odot_N(p8)$$





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Properties

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Existence

Lemma 2 (Existence of t-Induced T-nets). *Let $N = (P, T, F)$ be a well-formed free-choice net. N is either a T-net or there exist at least two different transitions $t_1, t_2 \in T$ such that $\boxdot_N(t_1)$ is proper and $\boxdot_N(t_2)$ is proper.*

Taking the complement can be repeated until T-net (there are always two options).

Lemma 3 (Existence of p-Induced P-nets). *Let $N = (P, T, F)$ be a well-formed free-choice net. N is either a P-net or there exist at least two different places $p_1, p_2 \in P$ such that $\odot_N(p_1)$ is proper and $\odot_N(p_2)$ is proper.*

Taking the complement can be repeated until P-net (there are always two options).

Well-formedness, liveness, boundedness, and free-choice are preserved

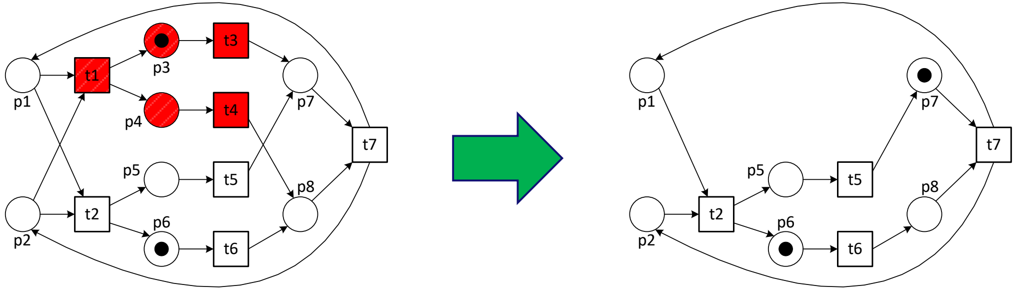
Lemma 4 (Well-Formedness of $\overline{N_{\boxdot(t)}}$). *Let $N = (P, T, F)$ be a well-formed free-choice net having a transition $t \in T$ such that $\boxdot_N(t)$ is proper. $\overline{N_{\boxdot(t)}} = (\overline{P}, \overline{T}, \overline{F})$ is the corresponding complement.*

(1) For any $\overline{M}, \overline{M}' \in \mathcal{B}(\overline{P})$, $\hat{M} \in \mathcal{B}(P)$, and $\sigma \in \overline{T}^*$: if $(\overline{N_{\boxdot(t)}}, \overline{M})[\sigma\rangle(\overline{N_{\boxdot(t)}}, \overline{M}')$, then $(N, \overline{M} \uplus \hat{M})[\sigma\rangle(N, \overline{M}' \uplus \hat{M})$.

(2) For any $M \in \mathcal{B}(P)$: if (N, M) is live and bounded, then $(\overline{N_{\boxdot(t)}}, mrk_{\boxdot}(N, t, M))$ is live and bounded.

(3) $\overline{N_{\boxdot(t)}}$ is well-formed and free-choice.

!



Well-formedness, liveness, boundedness, and free-choice are preserved

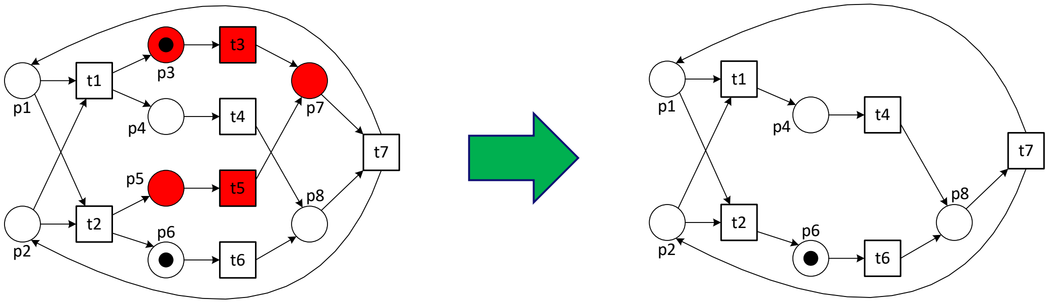
Lemma 5 (Well-Formedness of $\overline{N_{\odot(p)}}$). *Let* $N = (P, T, F)$ *be a well-formed free-choice net having a place* $p \in P$ *such that* $\odot_N(p)$ *is proper.* $\overline{N_{\odot(p)}} = (\overline{P}, \overline{T}, \overline{F})$ *is the corresponding complement.*

(1) *For any* $M, M' \in \mathcal{B}(P)$ *and* $\sigma \in T^*$*: if* $(N, M)[\sigma\rangle(N, M')$*, then* $(\overline{N_{\odot(p)}}, mrk_{\odot}(N, p, M))[\sigma\restriction_{\overline{T}}\rangle(\overline{N_{\odot(p)}}, mrk_{\odot}(N, p, M'))$.

(2) $\overline{N_{\odot(p)}}$ *is well-formed and free-choice.*

(3) *For any* $M \in \mathcal{B}(P)$*: if* (N, M) *is live and bounded, then* $(\overline{N_{\odot(p)}}, mrk_{\odot}(N, p, M))$ *is live and bounded.*

!



Reduction: Applying in sequence

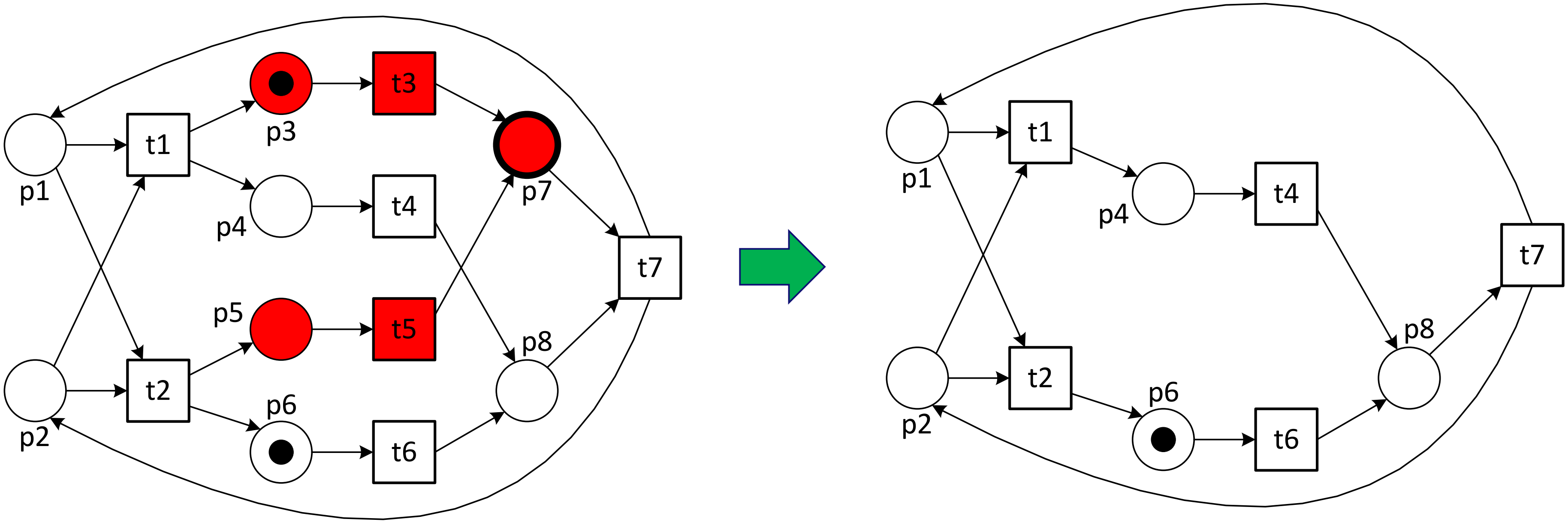
Definition 13 (Reductions). *Let* $N = (P, T, F)$ *be a well-formed free-choice net. A reduction of* N *is a sequence* $\gamma = \langle x^1, x^2, \ldots, x^n \rangle \in (P \cup T)^*$ *such that there exists a sequence of Petri nets denoted* $nets_N(\gamma) = \langle N^0, N^1, \ldots, N^n \rangle$ *where* $N^0 = N$*, and for any* $i \in \{1, \ldots, n\}$*:*

- $\boxdot_{N^{i-1}}(x^i)$ *is a proper* x^i*-induced T-net and* $N^i = \overline{N^{i-1}}_{\boxdot(x^i)}$ *if* $x^i \in T$.
- $\boxdot_{N^{i-1}}(x^i)$ *is a proper* x^i*-induced P-net and* $N^i = \overline{N^{i-1}}_{\odot(x^i)}$ *if* $x^i \in P$.

Definition 14 (Complete, T-, and P-Reductions). *Let* $N = (P, T, F)$ *be a well-formed free-choice net having a reduction* $\gamma = \langle x^1, x^2, \ldots, x^n \rangle \in (P \cup T)^*$ *with the corresponding sequence of Petri nets:* $nets_N(\gamma) = \langle N^0, N^1, \ldots, N^n \rangle$.[2]

- γ *is* x*-preserving if* $x \in P \cup T$ *is a place/transition in the remaining net* N^n.
- γ *is a complete reduction if* N^n *is a T-net or a P-net.*
- γ *is a T-reduction if* $\{x^1, x^2, \ldots, x^n\} \subseteq T$ *and* N^n *is a T-net.*
- γ *is a P-reduction if* $\{x^1, x^2, \ldots, x^n\} \subseteq P$ *and* N^n *is a P-net.*

Step 1



p1
t1
p3
t3
p7
p4
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p5
t5
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p4
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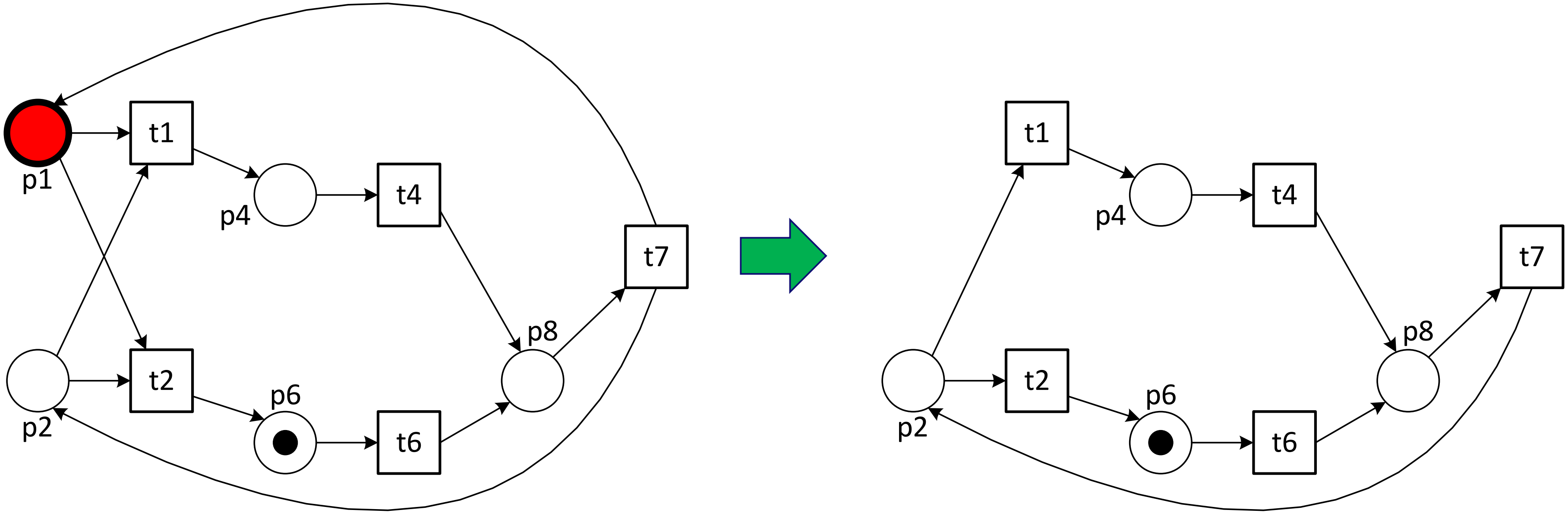
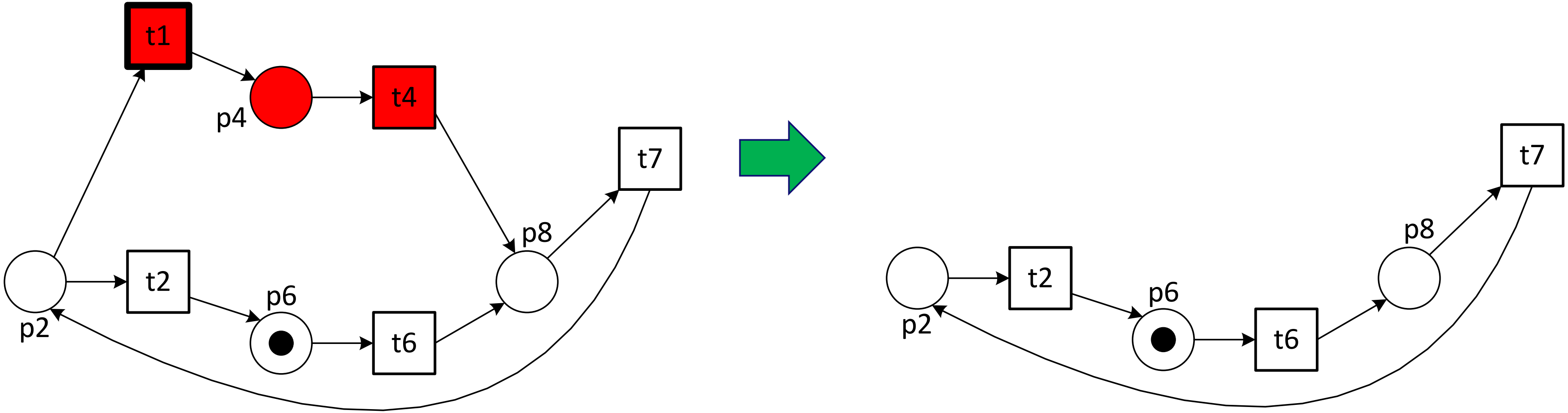
t2
p6
t6

Step 2



Step 3



Summary

- Any well-formed free-choice net can be turned into a well-formed T-net by a T-reduction.
- Any well-formed free-choice net can be turned into a well-formed P-net by a P-reduction.
- Subtle details:
  - If a reduction step is possible, there is a choice.
  - The corresponding markings preserve liveness and boundedness (see paper).

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Application to Perpetuality and Lucency

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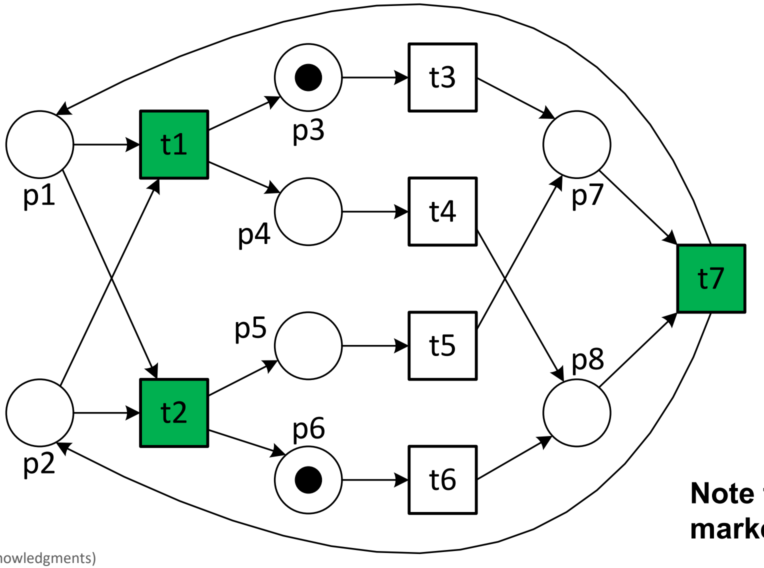
Regeneration Transition ⇒ Perpetual

Definition 16 (Regeneration Transitions). *Let Petri net $N = (P, T, F)$ be a Petri net. Transition $t_r \in T$ is a* regeneration transition *of N if the marked Petri net $(N, [p \in \bullet t_r])$ is live and bounded.*



A Petri net is perpetual if there exists at least one regeneration transition (independent of marking).

Note that this does not require a marked Petri net (related to next slide).

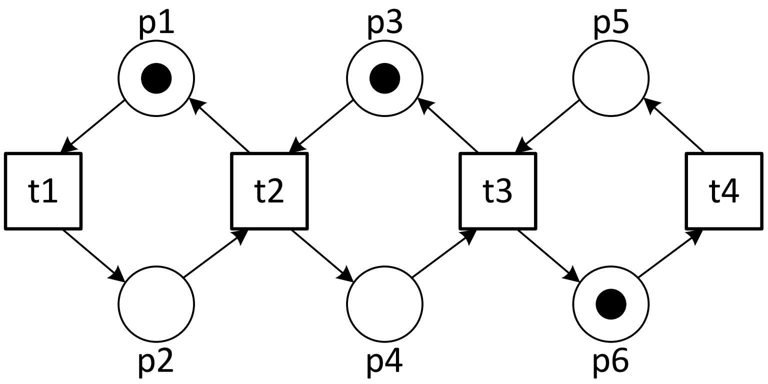
Perpetuality implies pc-safe

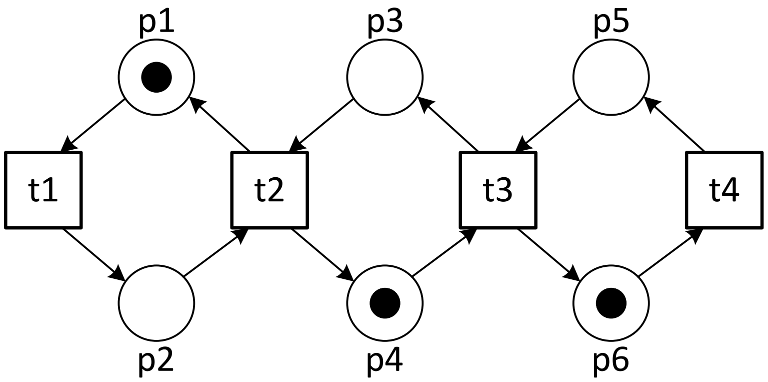
Definition 18 (PC-Safely Marked Nets). *Let Petri net* $N = (P, T, F)$ *be a Petri net.* $M \in \mathcal{B}(P)$ *is a* pc-safe *marking of* N *if for any* $X \in PComp(N)$: $M(X \cap P) = 1$, *i.e., each P-component contains precisely one token.* (N, M) *is a pc-safely marked net if* M *is a pc-safe marking of* N.

Lemma 7 (Perpetual Nets Are PC-Safely Marked). *Let* $N = (P, T, F)$ *be a perpetual well-formed free-choice net with regeneration transition* $t_r \in T$. *For any marking* $M \in \mathcal{B}(P)$: M *is pc-safe if and only if* $[p \in \bullet t_r] \in R(N, M)$.

Lucency

Definition 19 (Lucency [2]). *Petri net* $N = (P, T, F)$ *is* lucent *if each pc-safe marking enables a unique set of transitions, i.e., for any two pc-safe markings* M_1 *and* M_2*: if* $en(N, M_1) = en(N, M_2)$*, then* $M_1 = M_2$*.*





Note that there are three p-components that are safe.

Not lucent, but also not perpetual!

Perpetuality and pc-safeness are preserved downstream!!!!

Theorem 4 (Invariant Downstream Properties). *Let* $N = (P, T, F)$ *be a well-formed free-choice net having a reduction* $\gamma = \langle x^1, x^2, \ldots, x^n \rangle$ *with the corresponding sequence of nets* $nets_N(\gamma) = \langle N^0, N^1, \ldots, N^n \rangle$.

(1) If $t_r \in T$ *is a regeneration transition of* N *(i.e.,* $(N, [p \in \bullet t_r])$ *is live and bounded) and* γ *is* t_r*-preserving, then* t_r *is a regeneration transition of all nets in* $nets_N(\gamma)$ *(i.e.,* $(N^i, [p \in \bullet t_r])$ *is live and bounded for any* $i \in \{0, \ldots, n\}$*).*[3]

(2) If (N, M) *is pc-safe, then all markings in* $mrks_{N,M}(\gamma)$ *are pc-safe.* **!**

(3) If N *is perpetual, then all nets in* $nets_N(\gamma)$ *are perpetual.*

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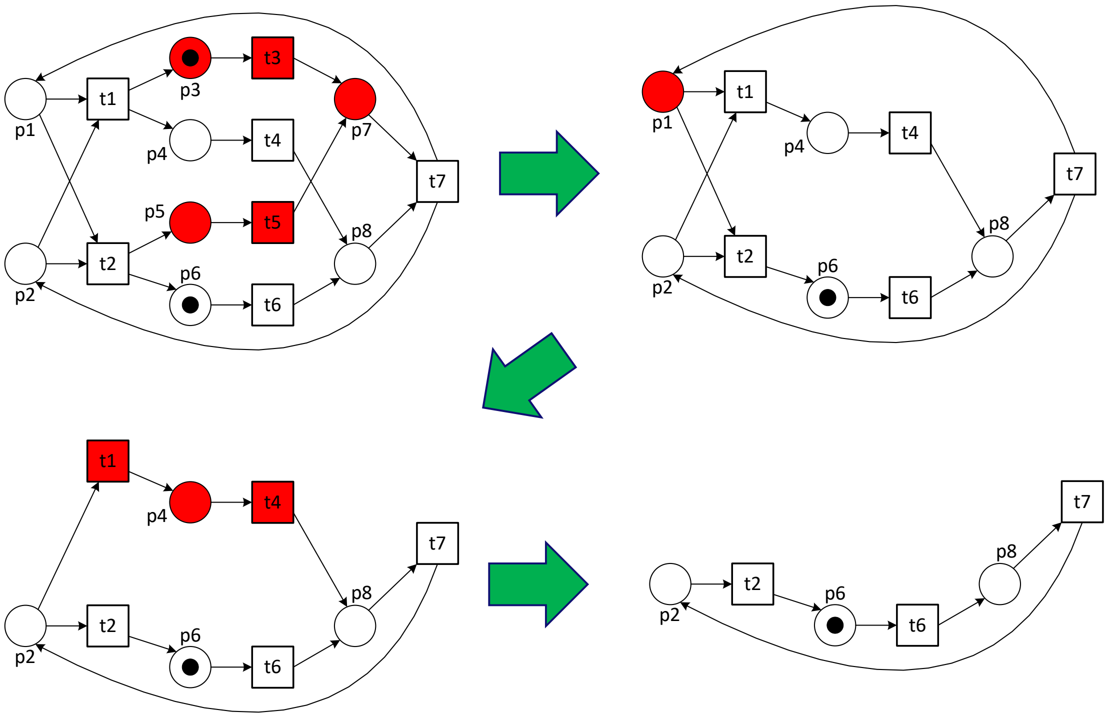
Perpetuality and pc-safeness are preserved downstream!!!!



Theorem 4 (Invariant Downstream Properties). *Let $N = (P, T, F)$ be a well-formed free-choice net having a reduction $\gamma = \langle x^1, x^2, \ldots, x^n \rangle$ with the corresponding sequence of nets $nets_N(\gamma) = \langle N^0, N^1, \ldots, N^n \rangle$.*

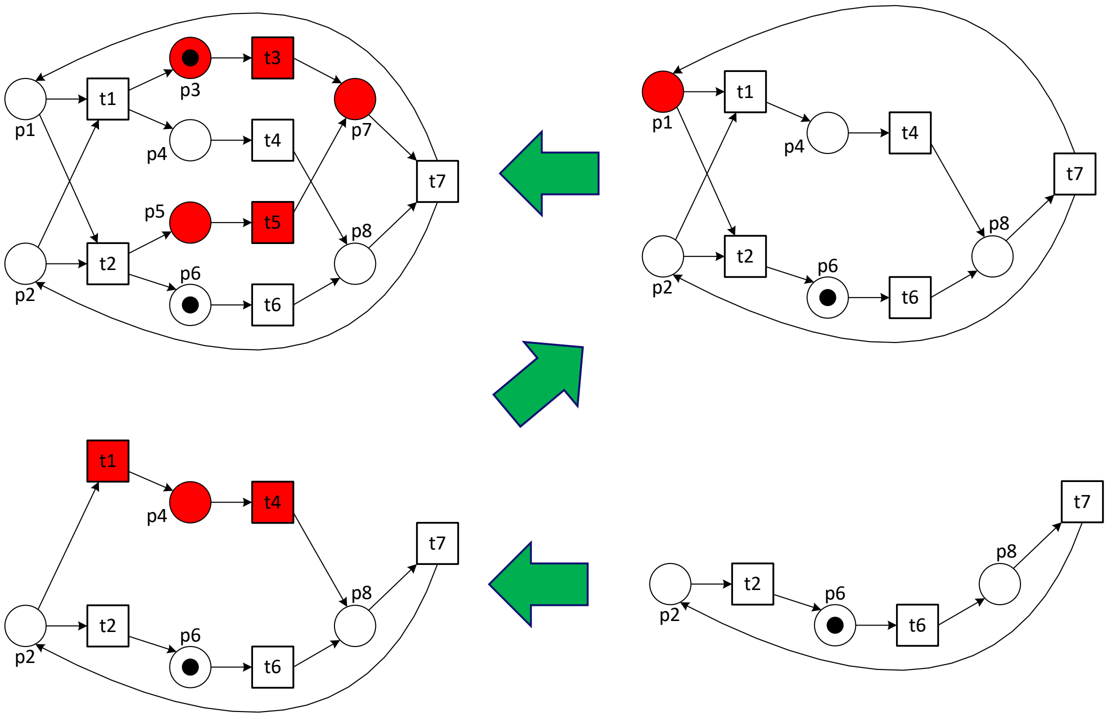
(1) If $t_r \in T$ is a regeneration transition of N (i.e., $(N, [p \in \bullet t_r])$ is live and bounded) and γ is t_r-preserving, then t_r is a regeneration transition of all nets in $nets_N(\gamma)$ (i.e., $(N^i, [p \in \bullet t_r])$ is live and bounded for any $i \in \{0, \ldots, n\}$).[3]

(2) If (N, M) is pc-safe, then all markings in $mrks_{N,M}(\gamma)$ are pc-safe.

(3) If N is perpetual, then all nets in $nets_N(\gamma)$ are perpetual.

Applies to any reduction!

Lucency is preserved upstream
(assuming perpetuality)



See the paper for the proof.

This implies that all perpetual well-formed free-choice nets are lucent.

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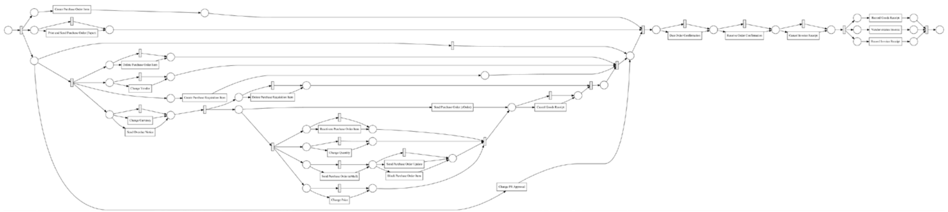
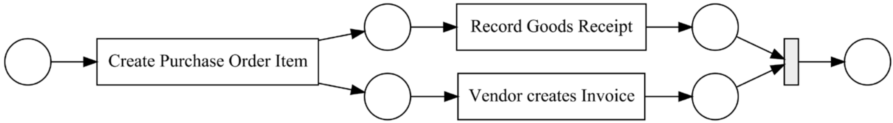
Framework based on reductions

short-circuit nets if workflow nets

(org)



(red)



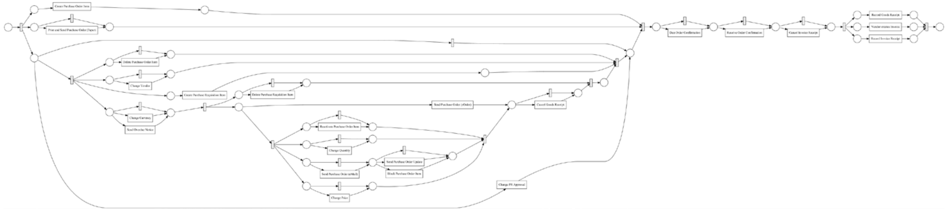
X is preserved downstream: If (org) is X, then (red) is X.

X is preserved upstream: If (red) is X, then (org) is X.

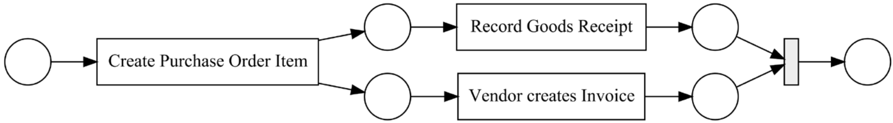
Framework based on reductions

short-circuit nets if workflow nets

(org)



(red)



Downstream: assume that (org) is well-formed and free-choice.

- **If (org) is pc-safe, then (red) is pc-safe.**
- **If (org) is perpetual, then (red) is perpetual.**

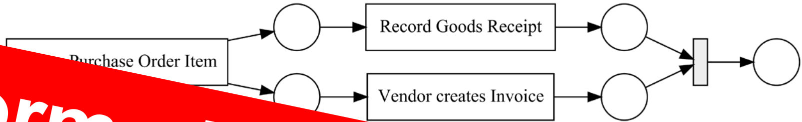
Upstream: assume that (org) is well-formed, free-choice, and perpetual.

- **If (red) is lucent, then (org) is lucent.**

Framework based on reductions

short-circuit nets if workflow nets

(red)



Since perpetual well-formed P-nets and T-nets are lucent, this implies that all perpetual well-formed free-choice nets are lucent!

Dow

- If (org)
- If (org) is perpetual,

Upstream: assume that (org) is well-formed, ... tual.

- If (red) is lucent, then (org) is lucent.

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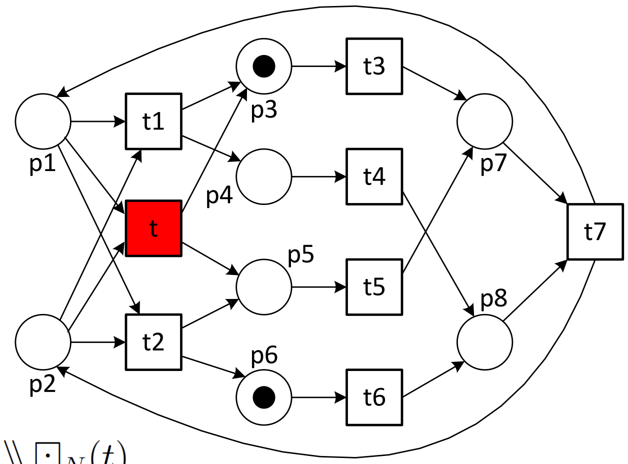
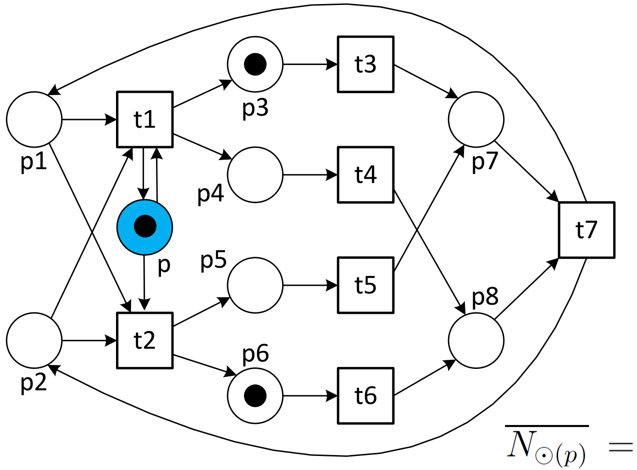
Limitations

- **Focus on well-formed free-choice nets.**
- **Not in reverse direction.**



$\overline{N_{\boxdot(t)}} = N \setminus\!\setminus \boxdot_N(t)$



$\overline{N_{\odot(p)}} = N \setminus\!\setminus \odot_N(p)$

Complement is well-formed and induced subnet is proper, but original is not.

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Related Work

Related Work (Chronological)

- Gérard Berthelot, Gérard Roucairol: Reduction of Petri-Nets. MFCS 1976: 202-209 (1976)
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- Joachim Wehler: Simplified proof of the blocking theorem for free-choice Petri nets. J. Comput. Syst. Sci. 76(7): 532-537 (2010)
- Wil van der Aalst: Process Mining - Data Science in Action, Second Edition. Springer 2016 (2016)
- Wil van der Aalst: Markings in Perpetual Free-Choice Nets Are Fully Characterized by Their Enabled Transitions. Petri Nets 2018: 315-336 (2018)
- Wil van der Aalst: Lucent Process Models and Translucent Event Logs. Fundam. Informaticae 169(1-2): 151-177 (2019)
- Wil van der Aalst: Reduction Using Induced Subnets To Systematically Prove Properties For Free-Choice Nets. CoRR abs/2106.03658 (2021)
- Wil van der Aalst: Free-Choice Nets With Home Clusters Are Lucent. CoRR abs/2106.03554 (2021). Accepted for FI.

reduction rules

free-choice theory

perpetuality

lucency

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An exciting new result!

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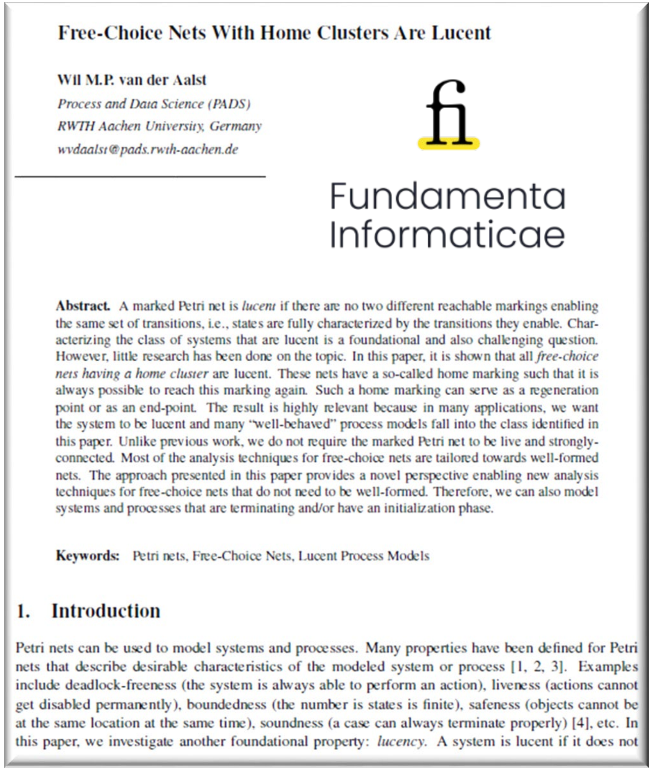
Free-Choice Nets With Home Clusters Are Lucent !!


Free-Choice Nets With Home Clusters Are Lucent

Wil M.P. van der Aalst
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Abstract. A marked Petri net is *lucent* if there are no two different reachable markings enabling the same set of transitions, i.e., states are fully characterized by the transitions they enable. Characterizing the class of systems that are lucent is a foundational and also challenging question. However, little research has been done on the topic. In this paper, it is shown that all *free-choice nets having a home cluster* are lucent. These nets have a so-called home marking such that it is always possible to reach this marking again. Such a home marking can serve as a regeneration point or as an end-point. The result is highly relevant because in many applications, we want the system to be lucent and many "well-behaved" process models fall into the class identified in this paper. Unlike previous work, we do not require the marked Petri net to be live and strongly-connected. Most of the analysis techniques for free-choice nets are tailored towards well-formed nets. The approach presented in this paper provides a novel perspective enabling new analysis techniques for free-choice nets that do not need to be well-formed. Therefore, we can also model systems and processes that are terminating and/or have an initialization phase.

Keywords: Petri nets, Free-Choice Nets, Lucent Process Models

1. **Introduction**

Petri nets can be used to model systems and processes. Many properties have been defined for Petri nets that describe desirable characteristics of the modeled system or process [1, 2, 3]. Examples include deadlock-freeness (the system is always able to perform an action), liveness (actions cannot get disabled permanently), boundedness (the number is states is finite), safeness (objects cannot be at the same location at the same time), soundness (a case can always terminate properly) [4], etc. In this paper, we investigate another foundational property: *lucency*. A system is lucent if it does not

- **Paper accepted for Fundamenta Informaticae (in print).**
- **Preprint: Wil van der Aalst: Free-Choice Nets With Home Clusters Are Lucent. CoRR abs/2106.03554 (2021).**
- **Completely different approach: Nets do not need to be well-formed.**
- **Main results:**
  - **Let (N,M) be a marked proper free-choice net having a home cluster. (N,M) is lucent.**
  - **The following problem is solvable in polynomial time: Given a marked proper free-choice net, to decide whether there is a home cluster.**

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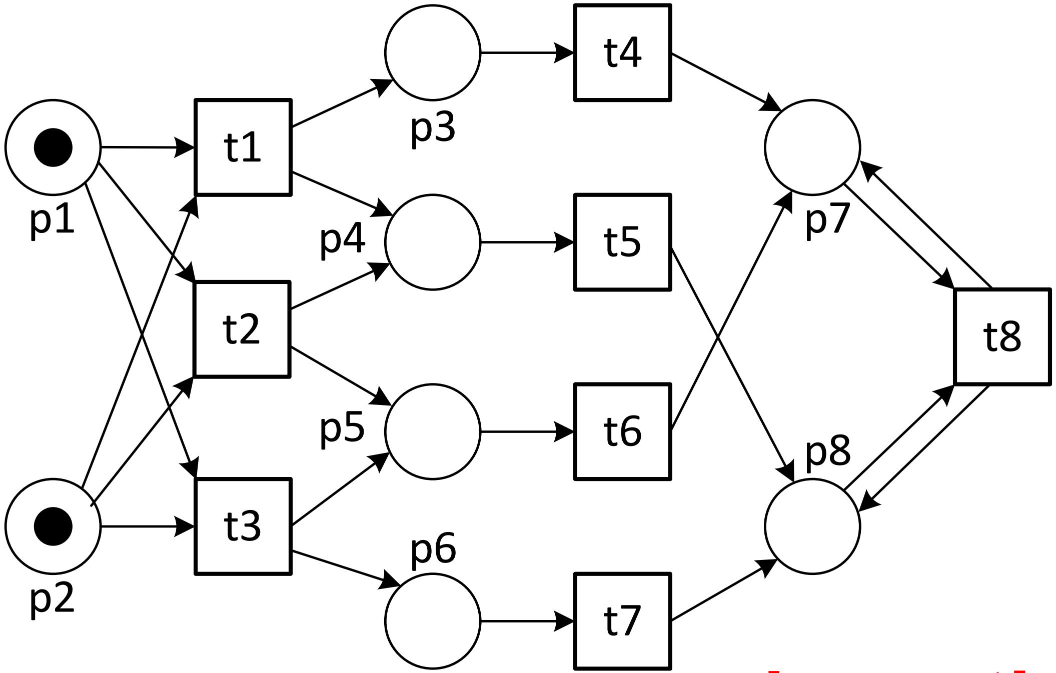
Example

Let (N,M) be a marked proper free-choice net having a home cluster. (N,M) is lucent.

- Proper (transitions have input and output places)
- [p7,p8] defines a home cluster (it is always possible to again mark just this cluster)

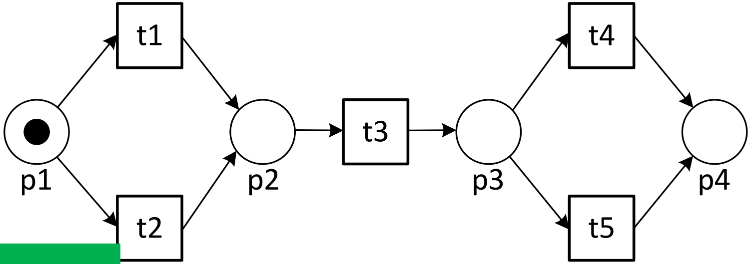


Lucent!

Example

Let (N,M) be a marked proper free-choice net having a home cluster. (N,M) is lucent.

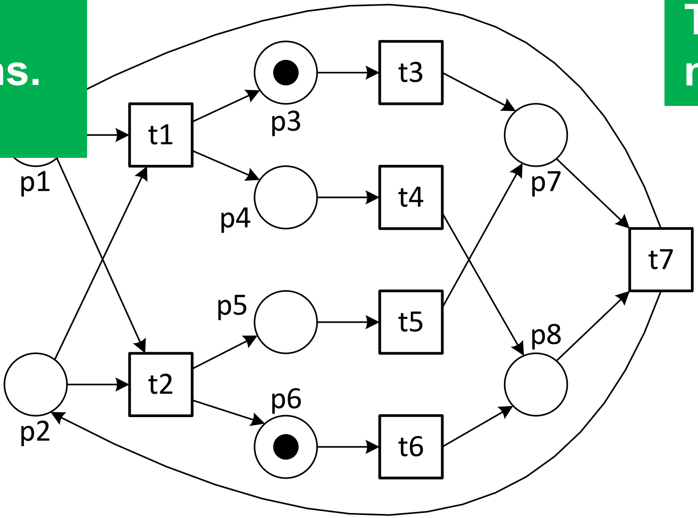
- Proper (transitions have input and output places)
- [p4] defines a home cluster (it is always possible to again mark just this cluster)



Lucent!

New tools (see paper):
- Expediting a transition.
- Rooted disentangled paths.
- Conflict-pairs.

The usual (indirect) machinery is not needed.



- Proper (transitions have input and output places)
- [p1,p2] defines a home cluster (it is always possible to again mark just this cluster)

Lucent!

Conclusion

Conclusion

- A framework for P- and T-reductions (strong properties for well-formed free-choice nets that can be reused in future proofs).
- Upstream and downstream preservation of properties.
- Example application: All perpetual well-formed free-choice nets are lucent.
- Can be also be used to prove the well-known blocking theorem.

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Future Work

- Build on the work for non-well-formed free choice nets! No need for the classical indirect machinery.
- The work on reductions was triggered by interactive/incremental process discovery.
- The work on proper free-choice nets having a home cluster was driven by searching for the right representational bias in process mining.
- Lucency is directly inspired process mining (see e.g., translucent event logs).

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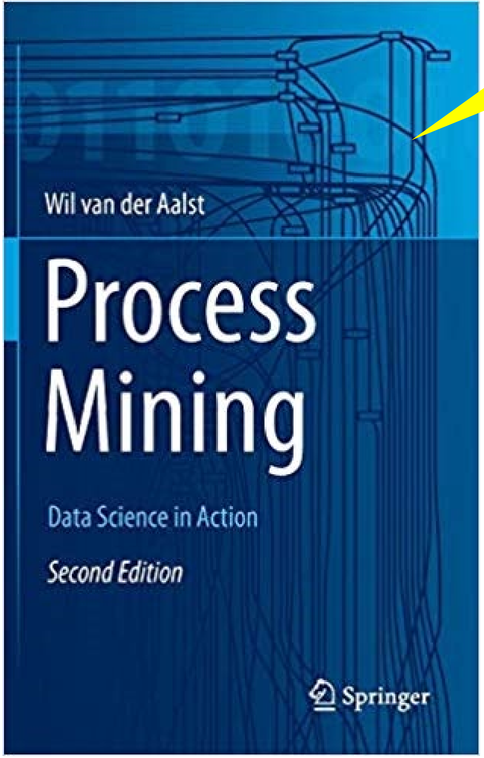
Learn more about process mining?

The killer application for Petri nets!



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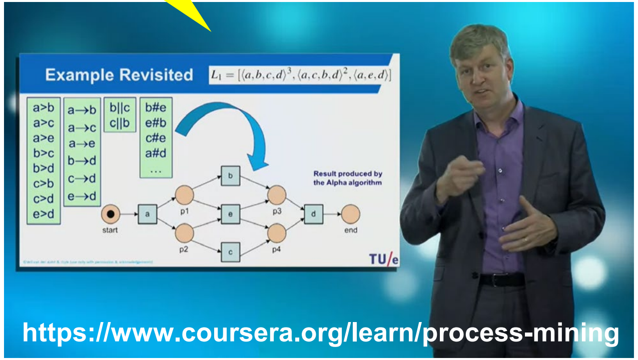

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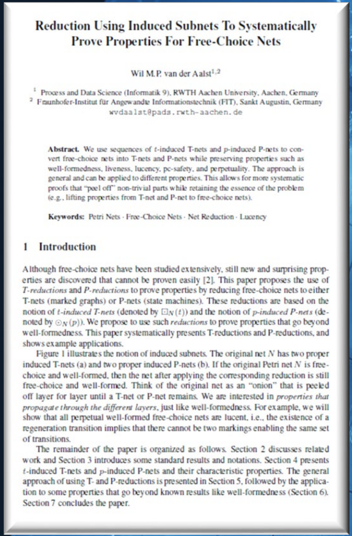
Petri Nets
Paris, 2021

Reduction Using Induced Subnets To Systematically Prove Properties For Free-Choice Nets



Reduction Using Induced Subnets To Systematically Prove Properties For Free-Choice Nets

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[2] Fraunhofer-Institut für Angewandte Informationstechnik (FIT), Sankt Augustin, Germany
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Abstract. We use sequences of *t*-induced T-nets and *p*-induced P-nets to convert free-choice nets into T-nets and P-nets while preserving properties such as well-formedness, liveness, lucency, pc-safety, and perpetuality. The approach is general and can be applied to different properties. This allows for more systematic proofs that "peel off" non-trivial parts while retaining the essence of the problem (e.g., lifting properties from T-net and P-net to free-choice nets).

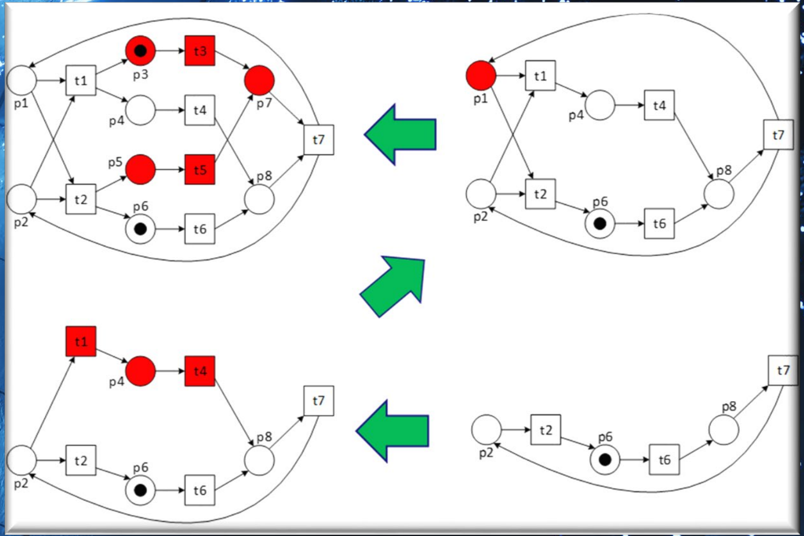
Keywords: Petri Nets · Free-Choice Nets · Net Reduction · Lucency

1 Introduction

Although free-choice nets have been studied extensively, still new and surprising properties are discovered that cannot be proven easily [2]. This paper proposes the use of *T-reductions* and *P-reductions* to prove properties by reducing free-choice nets to either T-nets (marked graphs) or P-nets (state machines). These reductions are based on the notion of *t-induced T-nets* (denoted by $\boxdot_N(t)$) and the notion of *p-induced P-nets* (denoted by $\odot_N(p)$). We propose to use such *reductions* to prove properties that go beyond well-formedness. This paper systematically presents T-reductions and P-reductions, and shows example applications.

Figure 1 illustrates the notion of induced subnets. The original net N has two proper induced T-nets (a) and two proper induced P-nets (b). If the original Petri net N is free-choice and well-formed, then the net after applying the corresponding reduction is still free-choice and well-formed. Think of the original net as an "onion" that is peeled off layer for layer until a T-net or P-net remains. We are interested in *properties that propagate through the different layers*, just like well-formedness. For example, we will show that all perpetual well-formed free-choice nets are lucent, i.e., the existence of a regeneration transition implies that there cannot be two markings enabling the same set of transitions.

The remainder of the paper is organized as follows. Section 2 discusses related work and Section 3 introduces some standard results and notations. Section 4 presents *t*-induced T-nets and *p*-induced P-nets and their characteristic properties. The general approach of using T- and P-reductions is presented in Section 5, followed by the application to some properties that go beyond known results like well-formedness (Section 6). Section 7 concludes the paper.



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